DNF => SOP
CNF => POS

$x \vee y \Rightarrow x + y$
$x \wedge y \Rightarrow x \cdot y$: logical multiplication

EECS 281: Homework #5 Due: Tuesday, October 26, 2004

Name: ______________________ Email: ______________________

0. Practice, study (do not hand in) Wakerly problems (solutions at www.wakerly.com): 4.13(a) and 4.13(b).

1a. Apply T13 & T4 to the expression $\overline{abc}$ resulting in: $\bar{a}+\bar{b}+\bar{c}$: $a+b+c$

1b. Apply T13' & T4 to the expression $\overline{a \vee \bar{b} \vee \bar{c}}$ resulting in: $\overline{a+\bar{b}+\bar{c}} = \bar{a}\cdot\bar{\bar{b}}\cdot\bar{\bar{c}} : \bar{a}\bullet b\bullet c$

1c. Apply T8 to the expression $(a \vee b)(a \vee \bar{b})$ resulting in (note: same as saying: $(a+b)(a+\bar{b})$:
$(a+b)(a+\bar{b}) = aa + a(b+\bar{b}) = a + a = a$

1d. Apply T8 to the expression $(a \vee b)(a \vee \bar{b})(\bar{a} \vee b)$ resulting in:
$(a+b)(a+\bar{b})(\bar{a}+b) = a(b+\bar{b})(\bar{a}+b) = (a)(\bar{a}+b) = a\bar{a} + ab = ab$

1e. Given the minterms $\sum_{abc}(1,2,4,7)$, write the DNF (i.e. SOP) expression:
$\bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$

1f. Give the maxterms of 1e: 0, 3, 5, 6

1g. Give the CNF (i.e. POS) expression of 1f (read Wakerly page 208):
$(a+b+c)\bullet(a+\bar{b}+\bar{c}) + (\bar{a}+b+\bar{c}) + (\bar{a}+\bar{b}+c)$

1h. Draw the logic gate schematic of 1g:

a
b
c

2a. Give the truth table, minterms, and Maxterms for the following function $f(a,b,c) = \overline{a\bar{b} \vee ac} \vee \bar{a}$;

$\bar{b}$	a b c	$a\bar{b}$	ac	$a\bar{b} \vee ac$	$\overline{a\bar{b} \vee ac}$	$\bar{a}$	f	minterms	maxterms	$b\bar{c}$	$\bar{a}+b\bar{c}$	$\bar{f}$
1	0 0 0	0	0	0	1	1	1			0	1	1
1	0 0 1	0	0	0	1	1	1			0	1	1
0	0 1 0	0	0	0	1	1	1			1	1	1
0	0 1 1	0	0	0	1	1	1			0	1	1
1	1 0 0	1	0	1	0	0	0			0	0	0
1	1 0 1	1	1	1	0	0	0			0	0	0
0	1 1 0	0	0	0	1	0	1			1	1	1
0	1 1 1	0	1	1	0	0	0			0	0	0

2b. Using boolean algebra (Wakerly page 199 Table 4-2, page 201, Table 4-3), give the DNF (i.e. SOP):

2b. Applying theorem T13' on $\overline{a\bar{b} \vee ac} \vee \bar{a}$, we now have $(\overline{a\bar{b}} \bullet \overline{ac}) + \bar{a}$

2b. Applying theorem T13, we now have $(\bar{a}+b)\cdot(\bar{a}+\bar{c}) + \bar{a}$

2b. Applying theorem T4, we now have $((\bar{a}+b)\cdot(\bar{a}+\bar{c})) + \bar{a} = (\bar{a}\bar{a} + \bar{a}c + b\bar{a} + b\bar{c})$

2b. Applying theorem T8', we now have $\bar{a}(b+\bar{c}) + \bar{a} + b\bar{c}$

a	b	c	$a\bar{b}$	ac	$a\bar{b}+ac$	$a\bar{b}+ac$	$\bar{a}$	f
0	0	0	0	0	0	1	1	1
0	0	1	0	0	0	1	1	1
0	1	0	0	0	0	1	1	1
0	1	1	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0
1	0	1	1	1	1	1	0	1
1	1	0	0	0	0			
1	1	1	0	1	1	0	0	0

$$\overline{a\bar{b}+ac} + \bar{a}$$

$$= (\overline{a\bar{b}} \cdot \overline{ac}) + \bar{a}$$

$$= [(\bar{a}+b)\cdot(\bar{a}+\bar{c})] + \bar{a}$$

$$= [\bar{a}(b+\bar{c})] + \bar{a} + b\bar{c}$$

$$= \bar{a}(b+\bar{c}) + \bar{a} + b\bar{c}$$

$$= (\bar{a}b+\bar{a}\bar{c}) + \bar{a} + b\bar{c}$$

$$= \bar{a}b + \bar{a}\bar{c} + \bar{a} + b\bar{c}$$

$$= \bar{a}(b+\bar{c}+1) + b\bar{c}$$

$$f = \bar{a} + b\bar{c}$$

$$\bar{a}\bar{a} + \bar{a}\bar{c} + b\bar{a} + b\bar{c}$$

$$\bar{a}(1+\bar{c}+b$$

2b. Factor out $\bar{a}$ and applying theorem T2, we now have $\bar{a}(1+b+\bar{c})+b\bar{c}$

2bbb. Applying theorem T1', we now have $\bar{a}+b\bar{c}$
(note: Does 2bbb match the truth table of 2a?)

$\bar{a}+b\bar{c} =) \quad \bar{a}(b+\bar{b}) + b\bar{c}(a+\bar{a})$

2c. Give the n-cubes for part 2bbb: $\bar{a}b+\bar{a}\bar{b} + ab\bar{c}+\bar{a}b\bar{c}$

$= \bar{a}bc+\bar{a}b\bar{c}+\bar{a}\bar{b}c+\bar{a}\bar{b}\bar{c}+ab\bar{c}+\bar{a}b\bar{c}$

2d. Give the minterms (i.e. 0-cubes or ON-set) for part 2bbb:

$\bar{a}b\bar{c}+ab\bar{c}+\bar{a}bc+\bar{a}\bar{b}c+\bar{a}\bar{b}\bar{c}$

2e. Did 2d match your truth table of 2a?

2f. Fill in the k-map from 2bbb, showing circles of only the terms of 2bbb:

$\therefore \Sigma(010, 110, 011, 001, 000)$

$\Sigma(2,6,3,1,0)$

	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$	1	1	1	1
a				1

2g. Give the optimal minimal SOP of 2f: $\bar{a}+b\bar{c}$

2h. Give the Maxterms (i.e. OFF-set) from part 2d: $\Pi(4,5,7)$

2i. Give the CNF of 2h (i.e. canonical product, POS, see Wakerly, page 208): Expression in POS form

3a. Show by circling in the k-map each term in function $f(a,b,c)=\bar{a}\bar{b} \vee \bar{b}c \vee ac \vee ab \vee \bar{c}\bar{b}$;

$(\bar{a}\bar{b}+\bar{b}c+ac+ab+\bar{c}\bar{b})$

$\bar{a}\bar{b}(c+\bar{c})+\bar{b}c(a+\bar{a}) + ac(b+\bar{b})+ab(c+\bar{c})+\bar{b}\bar{c}(a+\bar{a})$

		$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
0	$\bar{a}$	1	1		
1	a	1	1	1	1

3b. Show the optimal mimimal circling in the k-map of 3a (Wakerly, page 223, Fig. 4-27, Fig. 4-29):

	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$	1	1		
a	1	1	1	1

$\bar{a}\bar{b}$ 00x

$\bar{b}c$ x01

$(\overline{\bar{a}b})$

3c. Give the Minimal SOP of the k-map: $a+\bar{b}$

3d. Give the CNF (i.e POS) of 3c: $(\overline{\bar{a}b})$

4a Show by circling in the k-map each term in function $f(a,b,c,d) = \bar{a}\bar{b}\bar{c} \vee \bar{b}c \vee bcd$;

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1	1	1	1
$\bar{a}b$			1	
ab			1	
$a\bar{b}$			1	1

bcd

$\bar{a}\bar{b}\bar{c}(d+\bar{d}) + \bar{b}c(a+\bar{a})$

$bcd(a+\bar{a})$

$\bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}\bar{d}$

$\Rightarrow abcd + \bar{a}bcd + a\bar{b}cd$

$+ a\bar{b}c\bar{d} + \bar{a}\bar{b}cd + \bar{a}bc\bar{d}$

$\bar{b}ca(d+\bar{d}) + \bar{b}c\bar{a}(d+\bar{d})$

$\bar{b}cad + \bar{b}ca\bar{d} + \bar{b}c\bar{a}d + \bar{b}c\bar{a}\bar{d}$

4b Show the optimal mimimal circling in the k-map of 4a:

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	1	1	1	1
$\bar{a}b$			1	
ab			1	
$a\bar{b}$			1	1

$= \Sigma(1111, 0111, 1011, 0001, 0000, 1010, 0011, 0110)$

$= \Sigma(15, 7, 11, 10, 3, 6, 0, 1)$

4c. Give the Minimal SOP of the k-map: $\bar{a}\bar{b} + cd + a\bar{b}c + \bar{b}c\bar{d}$

4d. Give the CNF (i.e POS) of 4c: $\Pi(2, 4, 5, 8, 9, 12, 13, 14)$

4e. Give the minterms of the k-map in 4a: ____________________

4f. Group the minterms of 4e by the number of 1's:

4g. Do the Quine-McCluskey Algorithm in 4f.

Covered in Class Thursday 10/28/04

http://bear.ces.cwru.edu/eecs_281/eecs_281_20041028b.jpg

5a Show the optimal k-map for $\sum_{a,b,c,d}(1,2,3,5,7)$:

	$\bar c\bar d$	$\bar c d$	cd	$c\bar d$
$\bar a\bar b$		1	1	1
$\bar a b$		1	1	
ab				
$a\bar b$				

a	b	c	d	
0	0	0	1	$(\bar a\bar b\bar c d)$
0	0	1	0	$(\bar a\bar b c\bar d)$
0	0	1	1	$(\bar a\bar b c d)$
0	1	0	1	$(\bar a b\bar c d)$
0	1	1	1	$(\bar a b c d)$

5b. Give the Minimal SOP of the k-map: $\bar a d + \bar a\bar b c = \bar a(\bar b c + d)$

5c. Given the don't cares (10,11,12,13,14,15), show the optimal k-map:

	$\bar c\bar d$	$\bar c d$	cd	$c\bar d$
$\bar a\bar b$		1	1	1
$\bar a b$		1	1	
ab	d	d	d	d
$a\bar b$			d	d

1010	$a\bar b c\bar d$
1011	$a\bar b c d$
1100	$ab\bar c\bar d$
1101	$ab\bar c d$
1110	$abc\bar d$
1111	$abcd$

5d. Give the Minimal SOP of the k-map: $\bar a d + cd + \bar b c\bar d + ac$

5e. Do the Quine-McCluskey Algorithm of 5a only (not 5c):

0: 1 0001, 2 0010
1: 3 0011, 5 0101
3: 7 0111

00–1, 0–01, 0–11, 01–1

0––1, 0––1, 0––1, 0––1 → $\bar a d$

0001, 0010, 0011, 0101, 0111

Solution:

1,3	00–1	1,3,5,7	0––1	1,3,5,7	0––1
1,5	0–01	1,5,3,7	0––1	2,3	001–
2,3	001–	2,3	001–		
3,7	0–11				
5,7	01–1				

$\bar a d + \bar a\bar b c$

6. A programmer as written the following C code fragment:

```
f=0;
if (a & b) {
    if (c) { f=1; }
}
else if (b | c) { f=1; }
```

a	b	c	f	
0	0	0	0	
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	0	
1	0	1	1	$a\bar{b}c$
1	1	0	0	
1	1	1	1	abc

6a. Give the truth table for the variable f (assume that a, b, c are boolean values only):

6b. Give the optimal k-map of 6a.

	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$		1	1	1
a		1	1	

6c. Give the optimal minimum SOP:

$(c + \bar{a}b)$

6d. Re-write as optimal C code:

f = [(~a & b) | c]