

DNF  $\Rightarrow$  SOP  
 CNF  $\Rightarrow$  POS

$X \vee Y \Rightarrow X + Y$  : arithmetic  
 $X \wedge Y \Rightarrow X \cdot Y$  : logical multiplication

**EECS 281: Homework #5**

**Due: Tuesday, October 26, 2004**

Name: \_\_\_\_\_

Email: \_\_\_\_\_

0. Practice, study (do not hand in) Wakerly problems (solutions at [www.wakerly.com](http://www.wakerly.com)): 4.13(a) and 4.13(b).

- 1a. Apply T13 & T4 to the expression  $\overline{a+b+c}$  resulting in:  $\overline{a+b+c} = \overline{a+b} \cdot \overline{c} = \overline{a} \cdot \overline{b} \cdot \overline{c}$
- 1b. Apply T13 & T4 to the expression  $\overline{a \vee b \vee c}$  resulting in:  $\overline{a+b+c} = \overline{a} \cdot \overline{b} \cdot \overline{c} = \overline{a} \cdot \overline{b} \cdot \overline{c}$
- 1c. Apply T8 to the expression  $(a \vee b)(a \vee \overline{b})$  resulting in (note: same as saying:  $(a+b)(a+\overline{b})$ ):  
 $(a+b)(a+\overline{b}) = a + a\overline{b} + ab + b\overline{b} = a + a\overline{b} + ab + 0 = a + a\overline{b} + ab = a + a = a$
- 1d. Apply T8 to the expression  $(a \vee b)(a \vee \overline{b})(\overline{a} \vee b)$  resulting in:  
 $(a+b)(a+\overline{b})(\overline{a}+b) = a(\overline{a}+b) + b(\overline{a}+b) = a\overline{a} + ab + b\overline{a} + b^2 = 0 + ab + b\overline{a} + b = ab + b\overline{a} + b = b(\overline{a} + a + 1) = b(1+1) = 2b$
- 1e. Given the minterms  $\sum_{abc}(1, 2, 4, 7)$ , write the DNF (i.e. SOP) expression:  
 $\overline{a}b\overline{c} + \overline{a}b\overline{c} + a\overline{b}\overline{c} + abc$
- 1f. Give the maxterms of 1e: 0, 3, 5, 6
- 1g. Give the CNF (i.e. POS) expression of 1f (read Wakerly page 208):  
 $(a+b+c) \cdot (a+\overline{b}+\overline{c}) + (a+\overline{b}+\overline{c}) + (\overline{a}+\overline{b}+c)$
- 1h. Draw the logic gate schematic of 1g:

$\begin{matrix} b \\ a \\ c \end{matrix}$

2a. Give the truth table, minterms, and Maxterms for the following function  $f(a, b, c) = \overline{ab \vee ac \vee a}$ ;

	a	b	c	$\overline{ab}$	$\overline{ac}$	$\overline{ab \vee ac}$	$\overline{a}$	f	minterms	maxterms
0	0	0	0	1	1	1	1	0		
1	0	0	1	1	0	0	1	0		
0	0	1	0	1	1	1	1	0		
1	0	1	1	1	0	0	1	0		
1	1	0	0	0	1	0	0	0		
0	1	0	1	0	0	0	1	0		
1	1	1	0	0	1	0	0	0		
0	1	1	1	0	0	0	0	0		

2b. Using boolean algebra (Wakerly page 199 Table 4-2, page 201, Table 4-3), give the DNF (i.e. SOP):

- 2b. Applying theorem T13' on  $\overline{ab \vee ac \vee a}$ , we now have  $(\overline{ab} \cdot \overline{ac}) + \overline{a}$
- 2b. Applying theorem T13, we now have  $(\overline{a+b}) \cdot (\overline{a+c}) + \overline{a}$
- 2b. Applying theorem T4, we now have  $((\overline{a+b}) \cdot (\overline{a+c})) + \overline{a} = (\overline{a}\overline{a} + \overline{a}\overline{c} + b\overline{a} + b\overline{c}) + \overline{a} = \overline{a}\overline{c} + b\overline{a} + b\overline{c} + \overline{a}$
- 2b. Applying theorem T8', we now have  $\overline{a}(b+\overline{c}) + \overline{a} + b\overline{c}$

a	b	c	$\bar{a}\bar{b}$	$ac$	$\bar{a}b + ac$	$\bar{a}b + ac$	$\bar{a}$	$f$
0	0	0	0	0	0	1	1	1
0	0	1	0	0	0	1	1	1
0	1	0	0	0	0	1	1	1
0	1	1	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0
1	0	1	1	1	1	1	0	1
1	1	0	0	0	0	0	0	0
1	1	1	0	1	1	0	0	0

$$\overline{a\bar{b} + ac} + \bar{a}$$

$$= (\overline{a\bar{b}} \cdot \overline{ac}) + \bar{a}$$

$$= \overline{(a+b) \cdot (a+c)} + \bar{a}$$

$$= \overline{a(b+c)} + \bar{a} + b\bar{c}$$

$$= \bar{a}(b+c) + \bar{a} + b\bar{c}$$

$$= (\bar{a}b + \bar{a}c) + \bar{a} + b\bar{c}$$

$$= \bar{a}b + \bar{a}c + \bar{a} + b\bar{c}$$

$$= \bar{a}(b+c+1) + b\bar{c}$$

$$f = \bar{a} + b\bar{c}$$

$$\bar{a}\bar{a} + \bar{a}\bar{c} + b\bar{a} + b\bar{c}$$

$$\bar{a}(1+\bar{c}+b)$$

2b. Factor out  $\bar{a}$  and applying theorem T2, we now have  $\bar{a}(1+b+\bar{c}) + b\bar{c}$

2bbb. Applying theorem T1', we now have  $\bar{a} + b\bar{c}$   
 (note: Does 2bbb match the truth table of 2a?)

2c. Give the n-cubes for part 2bbb:  $\bar{a} + b\bar{c} = \bar{a}(cb+\bar{b}) + b\bar{c}(ca+\bar{a})$   
 $= \bar{a}b + \bar{a}\bar{b} + ab\bar{c} + \bar{a}b\bar{c}$   
 $= \bar{a}b\bar{c} + \bar{a}b\bar{c} + \bar{a}b\bar{c} + \bar{a}b\bar{c} + ab\bar{c}$   
 $+ \bar{a}b\bar{c}$

2d. Give the minterms (i.e. 0-cubes or ON-set) for part 2bbb:  $\bar{a}b\bar{c} + ab\bar{c} + \bar{a}b\bar{c} + \bar{a}b\bar{c}$

2e. Did 2d match your truth table of 2a? \_\_\_\_\_

2f. Fill in the k-map from 2bbb, showing circles of only the terms of 2bbb:

	$\bar{b}\bar{c}$	$\bar{b}c$	$bc$	$b\bar{c}$
$\bar{a}$	1	1	1	1
$a$				

2g. Give the optimal minimal SOP of 2f:  $\bar{a} + b\bar{c}$

2h. Give the Maxterms (i.e. OFF-set) from part 2d:  $\pi(4, 5, 7)$

2i. Give the CNF of 2h (i.e. canonical product, POS, see Wakerly, page 208):  $\downarrow$  Expression in pos form

3a. Show by circling in the k-map each term in function  $f(a, b, c) = \bar{a}\bar{b} \vee \bar{b}c \vee ac \vee ab \vee \bar{a}b$ :  
 $\bar{a}\bar{b}(c+\bar{c}) + \bar{b}c(ca+\bar{a}) + ac(cb+\bar{b}) + ab(c+\bar{c}) + \bar{b}\bar{c}(a+\bar{a})$   
 $(\bar{a}\bar{b} + \bar{b}c + ac + ab + \bar{c}\bar{b})$

	$\bar{b}\bar{c}$	$\bar{b}c$	$bc$	$b\bar{c}$
0 $\bar{a}$	1	1	1	1
1 $a$	1	1	1	1

3b. Show the optimal minimal circling in the k-map of 3a (Wakerly, page 223, Fig. 4-27, Fig. 4-29):

	$\bar{b}\bar{c}$	$\bar{b}c$	$bc$	$b\bar{c}$
$\bar{a}$	1	1	1	1
$a$	1	1	1	1

3c. Give the Minimal SOP of the k-map:  $a + \bar{b}$

3d. Give the CNF (i.e POS) of 3c:  $(\bar{a}b)$

$\bar{a}b\bar{c} + ab\bar{c} + \bar{a}b\bar{c} + \bar{a}b\bar{c}$   
 $\Sigma(010, 110, 011, 001, 000)$   
 $\Sigma(2, 6, 3, 1, 0)$

$\bar{a}\bar{b}$  00x  
 $\bar{b}c$  x01  
 $(\bar{a}b)$

4a Show by circling in the k-map each term in function  $f(a, b, c, d) = \bar{a}\bar{b}\bar{c} \vee \bar{b}c \vee bcd$ ;

	$\bar{c}\bar{d}$	$\bar{c}d$	$cd$	$c\bar{d}$
$\bar{a}\bar{b}$	1	1	1	1
$\bar{a}b$				
$ab$				
$a\bar{b}$				

Handwritten notes for 4a:

$$\bar{a}\bar{b}\bar{c}(d+\bar{d}) + \bar{b}c(a+\bar{a})$$

$$\Rightarrow \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{b}cd + \bar{b}c\bar{a}$$

$$\Rightarrow \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}\bar{c}\bar{d} + a\bar{b}cd + \bar{b}ca(d+\bar{d}) + \bar{b}c\bar{a}(d+\bar{d})$$

4b Show the optimal minimal circling in the k-map of 4a:

	$\bar{c}\bar{d}$	$\bar{c}d$	$cd$	$c\bar{d}$
$\bar{a}\bar{b}$	1	1	1	1
$\bar{a}b$				
$ab$				
$a\bar{b}$				

Handwritten notes for 4b:

$$= \Sigma(1111, 0111, 1011, 0001, 0000, 010, 0011, 0110)$$

$$= \Sigma(15, 7, 11, 10, 3, 6, 0, 1)$$

$$= \bar{b}cad + \bar{b}c\bar{a}\bar{d} + \bar{b}c\bar{a}d + \bar{b}c\bar{a}\bar{d}$$

4c. Give the Minimal SOP of the k-map:  $\bar{a}\bar{b} + cd + a\bar{b}c + \bar{b}c\bar{d}$

4d. Give the CNF (i.e POS) of 4c:  $\Pi(2, 4, 5, 8, 9, 12, 13, 14)$

4e. Give the minterms of the k-map in 4a: \_\_\_\_\_

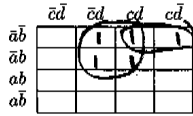
4f. Group the minterms of 4e by the number of 1's: \_\_\_\_\_

4g. Do the Quine-McCluskey Algorithm in 4f.

Covered in Class Thursday 10/28/04

[http://bear.ces.cwru.edu/eecs\\_281/eecs\\_281\\_20041028b.jpg](http://bear.ces.cwru.edu/eecs_281/eecs_281_20041028b.jpg)

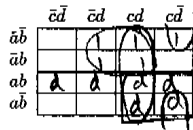
5a Show the optimal k-map for  $\Sigma_{a,b,c,d}(1,2,3,5,7)$ :



a	b	c	d	
0	0	0	1	( $\bar{a}\bar{b}\bar{c}d$ )
0	0	1	0	( $\bar{a}\bar{b}c\bar{d}$ )
0	0	1	1	( $\bar{a}\bar{b}cd$ )
0	1	0	1	( $\bar{a}b\bar{c}d$ )
0	1	1	1	( $\bar{a}bcd$ )

5b. Give the Minimal SOP of the k-map:  $\bar{a}d + \bar{a}bc = \bar{a}(\bar{b}c + d)$

5c. Given the don't cares (10,11,12,13,14,15), show the optimal k-map:



1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	
a	b	c	d	
$\bar{a}$	$\bar{b}$	$\bar{c}$	$\bar{d}$	
$\bar{a}$	$\bar{b}$	$\bar{c}$	d	
$\bar{a}$	$\bar{b}$	c	$\bar{d}$	
$\bar{a}$	$\bar{b}$	c	d	
$\bar{a}$	b	$\bar{c}$	$\bar{d}$	
$\bar{a}$	b	$\bar{c}$	d	
$\bar{a}$	b	c	$\bar{d}$	
$\bar{a}$	b	c	d	

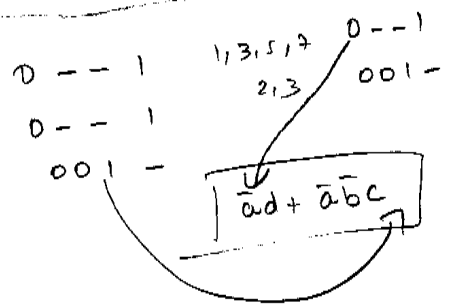
5d. Give the Minimal SOP of the k-map:  $\bar{a}d + cd + \bar{b}c\bar{d} + ac$

5e. Do the Quine-McCluskey Algorithm of 5a only (not 5c):

1	0001	00-1	0--1	0001
2	0010	0-01	0--1	0010
3	0011	0-11	$\bar{a}d$	0011
2	0101	01-1	0--1	0101
3	0111			0111

Solution:

1,3	00-1	1,3,5,7
1,5	0-01	1,5,3,7
2,3	001-	2,3
2,7	0-11	
5,7	01-1	



6. A programmer has written the following C code fragment:

```
f=0;
if (a & b) {
    if (c) { f=1; }
}
else if (b | c) { f=1; }
```

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- $\bar{a}bc$
- $\bar{a}b\bar{c}$
- $\bar{a}bc$
- $a\bar{b}c$
- $abc$

6a. Give the truth table for the variable f (assume that a, b, c are boolean values only):

6b. Give the optimal k-map of 6a.

	$\bar{b}\bar{c}$	$\bar{b}c$	$b\bar{c}$	$bc$
$\bar{a}$	0	1	1	1
a	0	1	1	1

6c. Give the optimal minimum SOP:

$$(c + \bar{a}b)$$

6d. Re-write as optimal C code:

$$f = [(a \& b) | c]$$