EECS 281: Name: Solutions **Test 3 (5 pages)** Email: ______ **Due: Thursday, April 7, 2005** Grade: ______ (100 points max)

Note: $\overline{ab} \neq \overline{a}\,\overline{b}$

$\overline{a+b} \neq \overline{a} + \overline{b}$

1. (10%) Please answer the following True or False in the context of Boolean Algebra:

T F $\prod_{ab}(3) = \overline{a\overline{b}}$ — F — $\prod_{ab}(3) = \overline{a+b} = \overline{a}\,\overline{b}$

T F $ab + abc = ab + abd$ — T — $\Rightarrow ab(1+c) = ab(1+d) \Rightarrow ab \cdot 1 = ab \cdot 1$

T F $\prod_{abc}(1,7,3,5,6) = \sum_{abc}(2,4,1,7,0)$ — F

T F $\sum_{ab}(3) = \overline{\overline{a}+\overline{b}}$ — T — $\Rightarrow \overline{\overline{a}+\overline{b}} \Rightarrow \overline{\overline{a}} \cdot \overline{\overline{b}} \Rightarrow ab$

T F $ab\overline{c} + abc = ab\overline{d} + abd$ — T — T8′ $\Rightarrow ab(\overline{c}+c) = ab(\overline{d}+d)$

T5′ $\Rightarrow ab \cdot 1 = ab \cdot 1$

2. (10%) Use Boolean Algebra to establish the identity. Show the Theorem numbers (i.e. T1-T13) for each step of your proof:

Theorem	Expression
	$1 = a(a+b) + (\overline{a}+b)(a+\overline{b}) + \overline{a\overline{b}}$
T8, T13	$= a + ab + (\overline{a}+b)(a+\overline{b}) + \overline{a} + b$
T7	$= (a+\overline{a}) + ab + (\overline{a}+b)(a+\overline{b}) + b$
T5	$= 1 + ab + (\overline{a}+b)(a+\overline{b}) + b$
T2	$= 1$

3a. (20%) Show the optimal minimal circling in the k-map in minterm function $\cancel{\Pi_{abcd}}$ Σ_{abcd} $(2, 3, 5, 7, 8, 9, 13, 15)$ in the left-hand figure below.

3a

	$\bar c\bar d$	$\bar c d$	cd	$c\bar d$
$\bar a\bar b$			1	1
$\bar a b$		1	1	
ab		1	1	
$a\bar b$	1	1		

Minimal k-map

3d

	$\bar c\bar d$	$\bar c d$	cd	$c\bar d$
$\bar a\bar b$			1	1
$\bar a b$		1	1	
ab		1	1	
$a\bar b$	1	1		

Static-1 hazard free k-map

3b. Give the MSOP in cube notation= 100X + X1X1 + 001X

3c. Give the MSOP in symbolic boolean algebra= $\bar a \bar b c + a \bar b \bar c + bd$

3d. Show the optimal k-map designed to cover static-1 hazards in the right-hand figure above.

4a. (20%) Show the optimal multi-output minimal circling the terms and in the k-map in minterm function $F = \sum_{abcd} = (2, 3, 5, 7, 8, 9, 13, 15)$ and $G = \sum_{abcd} = (2, 3, 4, 6, 8, 9, 12, 14)$. Indicate which circle belongs to what function.

F

	$\bar c\bar d$	$\bar c d$	cd	$c\bar d$
$\bar a\bar b$			1	1
$\bar a b$		1	1	
ab		1	1	
$a\bar b$	1	1		

G

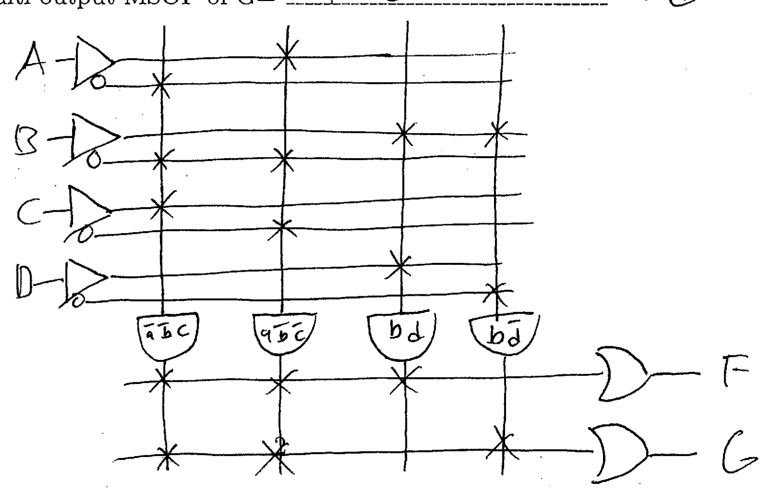
	$\bar c\bar d$	$\bar c d$	cd	$c\bar d$
$\bar a\bar b$			1	1
$\bar a b$	1			1
ab	1			1
$a\bar b$	1	1		

4b. Give the boolean algebra common term of multi-output MSOP = $\bar a \bar b c + a \bar b \bar c$

4c. Give the boolean algebra multi-output MSOP of F= $a \bar b \bar c + bd + \bar a \bar b c$

4d. Give the boolean algebra multi-output MSOP of G= $a \bar b \bar c + \bar a \bar b c + b \bar d$

4e. Draw and fill in the PLA:



5a. (20%). Do the Quine-McCluskey Algorithm of $\sum_{a,b,c,d}(2,3,5,7,8,9,13,15)$.

Group	Minterms	0-cubes	match?	Minterms	1-cubes	match?	Minterms	2-cubes
G_0								
G_1	2 8	0010 1000	✓ ✓	(2,3) (8,9)	001X 100X			
G_2	3 5 9	0011 0101 1001	✓ ✓ ✓	(3,7) (5,7) (5,13) (9,13)	0X11 01X1 X101 1X01	 ✓ ✓ 	(5,7,13,15) (5,7,13,15)	X1X1 X1X1
G_3	7 13	0111 1101	✓ ✓	(7,15) (13,15)	X111 11X1	✓ ✓		
G_4	15	1111	✓					

5b. Fill in the covering table

EPI?	PI-cubes	2	3	5	7	8	9	13	15		
✓	X1X1			✓	✓			✓	✓		
✓	001X	✓	✓								
✓	100X					✓	✓				
	0X11		✓		✓						
	1X01						✓	✓			
	~~[illegible]~~										
	Covered?	✓	✓	✓	✓	✓	✓	✓	✓		

5c. Give the boolean algebra MSOP= $bd + \bar{a}\bar{b}c + a\bar{b}\bar{c}$

6a. (10%) Given $\sum_{a,b,c,d}(2,3,5,7,8,9,13,15)$ and the don't cares (1,11,14), show the optimal k-map:

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$		X	1	1
$\bar{a}b$		1	1	
ab		1	1	X
$a\bar{b}$	1	1	X	

6b. Give the boolean algebra MSOP of the k-map: $d + \bar{a}\bar{b}c + a\bar{b}\bar{c}$

7. (10%) A programmer has written the following C code fragment (assume variables are 1-bit):

```
f=0;
if ( (a | b) & c){
     if (b) { f=1; }
}
else if (a & b) { f=0; }
```

7a. Give the truth table for the variable f (assume that a, b, c are boolean values only):

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

7b. Give the optimal k-map of 7a.

	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
$\bar{a}$			1	
a			1	

7c. Give the boolean algebra MSOP of the k-map: bc

7d. Re-write as optimal C code:

f = b & c

= (b & c) ? 1:0;

$t_n = 2t_{n-1} + n - 1;$

x1. (5% Extra credit) Write the C language for-loop for the recurrence equation, $t_n = 2t_{\cancel{n}} + n - 1$, where $t_0 = 2$.

```
int i, n, t=2;
                                       tn     2tn-1
for(i=1; i!=n+1; i++) { t = (t<<1) + i - 1; }
```

x2. (10% Extra credit) Write the 8051 assembler for the recurrence equation of problem x1, use R0 for variable i, R1 for variable n, R2 for variable t.

```
      mov  R2, #2    ; t=2
      mov  R0, #1    ; i=1
for:  mov  A, R1     ; n
      inc  A         ; n+1
      xrl  A, R0     ; if (n+1==i)
      jz   fdone     ; then goto fdone
      mov  A, R2     ; tn-1
      add  A, R2     ; 2tn-1
      add  A, R0     ; 2tn-1 + i
      dec  A         ; 2tn-1 + i - 1
      mov  R2, A     ; tn
      inc  R0        ; i++
      ajmp for
fdone: ...
```

Theorem	Relationship	Dual	XOR	Property
T1	$a1 = a$	$a + 0 = a$	$a \oplus 0 = a$	Identity
T2	$a0 = 0$	$a + 1 = 1$	$a \oplus 1 = \overline{a}$	Domination
T3	$aa = a$	$a + a = a$	$a \oplus a = 0$ $a \oplus a \oplus a = a$	Idempotency
T4	$\overline{\overline{a}}$			Involution
T5	$a\overline{a} = 0$	$a + \overline{a} = 1$	$a \oplus \overline{a} = 1$	Complement
T6	$ab = ba$	$a + b = b + a$	$a \oplus b = b \oplus a$	Commutative
T7	$(ab)c = a(bc)$	$(a+b)+c = a+(b+c)$	$(a \oplus b) \oplus c = a \oplus (b \oplus c)$	Associative
T8	$(a+b)(a+c) = a + bc$	$a(b+c) = ab + ac$	$a(b \oplus c) = ab \oplus ac$	Distributive
T9	$a(a+b) = a$	$a + ab = a$	$a \oplus ab = a\overline{b}$	Absorption Covering
T10	$(a+b)(a+\overline{b}) = a$	$ab + a\overline{b} = a$	$ab \oplus a\overline{b} = a$	Combining
T11	$(a+b)(\overline{a}+c)(b+c) = (a+b)(\overline{a}+c)$	$ab + \overline{a}c + bc = ab + \overline{a}c$		Consensus Proof by k-map
T12	$a + a + \cdots + a = a$	$aa \cdots a = a$	$a \oplus a \oplus \cdots \oplus a_{odd} = a$ $a \oplus a \oplus \cdots \oplus a_{even} = 0$	Generalized Idempotency
T13	$\overline{a+b} = \overline{a}\overline{b}$	$\overline{ab} = \overline{a} + \overline{b}$	$\overline{ab} = \overline{a} \oplus \overline{b} \oplus \overline{a}\overline{b}$	DeMorgan
XOR	$ab = a \oplus \overline{b} \oplus \overline{a}\overline{b}$	$a + b = a \oplus b \oplus ab$	$a \oplus b = \overline{a} \oplus \overline{b} = a\overline{b} + \overline{a}b$	Definition